We study a special bilevel programming problem that arises in transactions between a Natural Gas Shipping Company (NGSC) and a Pipeline Operator (OP) when facilities of the latter are used by the former. Because of the business relationships between these two actors, the timing and objectives of their decision-making process are different and sometimes even opposed. In order to model that, bilevel programming was traditionally used in previous works. Later, the problem was expanded and theoretically studied to facilitate its solution; this included extension of the upper level objective function, linear reformulation, heuristic approaches, and branch-and-bound techniques. In this paper, we present a linear programming reformulation of the latest version of the model, which is significantly faster to solve when implemented computationally. Even more, this new formulation makes it easier to theoretically analyze the problem, allowing us to draw some conclusions about the nature of the solution of the modified problem. Numerical results concerning the running time, convergence, and optimal values, are presented and compared to previous reports, showing a significant improvement in speed without sacrificing the solution’s quality.

Furthermore, since elements of uncertainty are definitely present in the bilevel natural gas cash-out problem, its stochastic formulation is developed in form of a bilevel multi-stage stochastic programming model with recourse. After reducing the original formulation to a bilevel linear problem, a stochastic scenario tree is defined by its node events, and time series forecasting is used to produce stochastic values for data of natural gas price and demand. Numerical experiments were run to compare the stochastic solution with the perfect information solution and the expected value solutions.

**Keywords:** bilevel programming, natural gas, cash-out problem, stochastic programming