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A stability analysis of the Nord Pool system using hourly spot price data

对采用按时现货价格数据的北欧电力库系统之稳定性分析

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Abstract - **Electricity prices are known to spike during peak hours, only to revert to normal levels during off-peak hours. We introduce a generalization of the time-varying independent spike model commonly used to model the electricity spot price from daily data to hourly data to cope with this feature.**

We let the probability of extreme prices depend on several variables, such as consumption, reserve margin or wind power. The model can then be used to forecast the risk of extreme prices.

More factors become relevant for predicting extreme events when moving to hourly data, but consumption is still the most important factor. The methodology is showcased by illustrating how extreme prices can be forecasted by predicting the consumption.

Keywords - **Electricity Market, HMM, Forward contract, EM-algorithm, Stability analysis.**

I. INTRODUCTION

Power generation has historically been operated according to some centralized designing. A vertical market structure was common, as that ensured that production plans would cover expected consumption and some extra margin, see [1]. Deregulation of the electricity market began in Chile in the early 1980s, whereupon many countries followed. A reasonably fair market means that prices will contain information about the current state of the market, cf. [2], and also information about the underlying power system, see [3]. One example of the quality of the information carried in markets is presented in [4] where they show that predictions markets generally outperformed election polls in nearly every U.S. Presidential elections between 1988 and 2004.

The electricity spot price is characterized by several features that are not common in other commodity prices. These include yearly, weekly and intra-day seasonality as well as extreme (very high and very low) prices. The latter are referred to as spikes and drops, see [5], and carry important information about the state of the physical power system, cf. [3, 6]. The price information was used to assess the stability in the Nord Pool power system using daily data under four different scenarios in [7]. They found that the consumption influenced both the probability of having spikes and drops, as well as the probability of reverting back to the normal conditions. It was expected that other variables, such as the reserve margin or wind power, also would influence those probabilities, cf. [8], but they found no statistical support for that hypothesis.

It can be speculated that variations in the wind power or reserve margin is operating on a shorter time scale than days, and that wind power therefore was deemed unnecessary in [7]. The purpose of this paper is to study the stability of the Nord Pool power system by using market data as proxy. It extends currently used models to cope with hourly data, in order to better understand the within day stability, and to explore if factors like wind power production and the reserve margin are relevant on a shorter time interval.

The remained of the paper is organized as follows. Section II discusses the electricity market and introduces a suitable statistical model. Estimation of the parameters is discussed in Section III while Section IV demonstrates how the methodology can forecast spikes. Section V concludes the paper.

II. A MODEL OF THE ELECTRICITY SPOT

The integration of renewable energy into the power system has made production planning much more complex, as we cannot know for sure what the production will be, cf. [9]. This complexity has resulted in volatility clustering and extreme prices in the electricity spot price, see [10] for an overview of stylized facts in the electricity spot price.

It is well known that the demand for electricity is inelastic, meaning that customers are rarely adopting their consumption according to the price, even though [11] and [12] presents strategies for changing this. The demand is varying on a yearly, weekly and daily scale, introducing seasonality in the price.

The yearly seasonality can be difficult to model, as it is due to physical processes that follow a cyclical pattern, but where the timing varies between years. That makes models that uses sums of trigonometric functions or wavelets, see [13], prone to overfitting the data. However, these techniques are still useful for modeling the weekly and daily seasonal patterns, as those patterns are constant over time and trigonometric methods do also work well when removing the seasonal component for a fixed set of data. It was noted in [14] that the spot and future prices are cointegrated, as their price implicitly depends on the same factors, a feature that we will use in this paper.

The extreme prices often cluster, as there could be several consecutive days with extreme prices before reverting to normal conditions. It was argued in [15] that this is best described by a Markov Regime Switching (MRS) model.

The electricity price is also known to be mean reverting, see [10], meaning that the price fluctuates around some equilibrium price, and will return to it even if some external disturbance caused causes a temporary deviation.

Our model belongs to the class of second generation Independent Spike Models, which is a special case of the Markov Regime Switching model, see [5, 16]. Early models used two regimes, but it is nowadays common to have three regimes, see [17]. These are the regime for high prices called spikes (S), the regime for low prices called drops (D) and a base regime (B) for normal prices. The transition between these is often assumed to be governed by a time invariant transition probability matrix, but recent studies indicate that this may be suboptimal, cf. [8, 7].

Allowing the transition probabilities to be time varying makes it possible to model the increasing probability for spikes when there is a shortage of electricity, and correspondingly to model the increasing probability for drops (and decreasing probability for spikes) when there is excess supply. The excess supply has even resulted in negative prices in both the German and Danish markets. Instead we parametrize the transition matrix according to

$$P(x_t) = \begin{bmatrix} p_{BB}(x_t) & p_{BS}(x_t) & p_{BD}(x_t) \\ p_{SB}(x_t) & p_{SS}(x_t) & 0 \\ p_{DB}(x_t) & 0 & p_{DD}(x_t) \end{bmatrix} \quad (1)$$

where the models has been restricted so that transitions directly from spikes (S) to drops (B) or vice versa are prohibited and $p_{BB}(x_t) = 1 - p_{BS}(x_t) - p_{BD}(x_t)$. Each probability (here we take $p_{BS}(x_t)$ as an example) is a function of an explanatory variable, given by

$$p_{BS}(x_t) = \frac{\exp(\beta_{BS,0} + \beta_{BS,1}x_t)}{1 + \exp(\beta_{BS,0} + \beta_{BS,1}x_t) + \exp(\beta_{BD,0} + \beta_{BD,1}x_t)}$$

This multinomial logistic mapping is common in regression problems, see [18] for details. The denominator ensures that all probabilities will be between zero and one while the numerator is made up of a base level $\beta_{BS,0}$ and a term that captures the influence of the explanatory variable $\beta_{BS,1}$. The change in probability when changing x_t is closely related to $\beta_{BS,1}$. We also tried quadratic forms and/or combinations of factors but found no convincing statistical support for any of these, see [19]. All explanatory variables was normalized according to

$$x_t = \frac{x_t}{\max_{u \in 1:T} x_u} \quad (2)$$

This does not change the model but it makes it easier to interpret and compare the estimates between different explanatory variables. Not scaling the variables would result in different estimates if the consumption was measured in MWh or GWh. The influence after scaling the variables varies between $\beta_{BS,0}$ when x_t is close to the smallest value and $\beta_{BS,0} + \beta_{BS,1}$ when x_t is at the largest value.

What remains is to specify the models for each regime. Most second generation Independent Spike Models use some forward contract as the yearly seasonal adjustment. It was noted in [14] that the spot price and forward price is cointegrated, meaning that the difference between them is a stationary process. It can be shown that the forward price F_n at time t_n is given by the discounted, risk-neutral conditional expectation of the average spot price

$$F_n = p(t_n, t_n + T) E^Q \left[\frac{1}{T} \int s(u) du \right] \quad (3)$$

where $s(\cdot)$ is the electricity spot price and $p(t_n, t_n + T)$ is a zero coupon bond with maturity T discounting the value back to time t_n. The spot price and the difference between the logarithm of the spot price $y_n = \log(s_n)$ and the logarithm of the one month ahead forward price $f_n = \log(F_n)$ are presented in Fig. 1, confirming the strong relation between the spot and forward. Daily and hourly effects were coped with by using dummy variables, cf. [20].

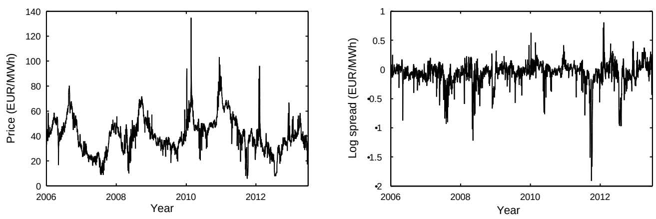

Fig 1. The spot price (left) and spread between the logarithm of the spot price and the logarithm of the one-month ahead forward price on Nord Pool (right) between 2006 and 2014.

The dynamics for the regimes are given by

$$y_{n+1} = \begin{cases} y_n + a(\mu_n - y_n) + \sigma y_n^{\gamma} z_n & if\ R_{n+1} = B \\ f_n + \xi^S & if\ R_{n+1} = S \\ f_n - \xi^D & if\ R_{n+1} = D \end{cases} \quad (4)$$

where R_n is a hidden Markov chain governing the state of the market. Similar models are used to describe the economy with booms and recessions. The mean reversion level $\mu_n = \eta f_n$ in the base regime is a factor compensating for the risk premium η times the logarithm of the month ahead forward, a and σ are positive constants while ξ^S and ξ^D are independent and identically distributed (*iid*) random variables having some known distribution (typically log-normal or Gamma), see [17] for various European markets. We take the risk premium as constant, even though [21] indicates that it may be related to the levels in the water reservoirs (a substantial part of the power traded at Nord Pool is generated in hydro power plants). However, we believe that this approximation is justified as the effect from misspecifying the mean is small compared to misspecifying the variance when it comes to the regime classification which is the primary purpose of the model.

III. Empirical Study

We have fitted several models to daily and hourly data, see [19] for details. The EM algorithm was used to optimize the log-likelihood function, cf. [17, 22]. The EM algorithm is often more robust than direct maximization of the likelihood function, see [18].

A general result that is valid across several markets and spike distributions was that there was no need for the CEV dynamics when introducing the time varying transition probabilities, γ=0. This is in line with the findings in [5].

The parameter estimates when using daily observations from Nord Pool between 2006 Q1 (or 2009 Q1) and the end of 2013 are presented in Table 1. The consumption and production are highly relevant variables, as noted in [7]. The reserve margin does on the other hand not significantly influence the probability of going from any state to any other state (the β_0 parameter provides an intercept, while the β_1 parameter gives the actual influence of the external variable), while wind power production helps a little, as more wind power increases the likelihood for reverting to the base regime when in the spike regime ($\beta_{SB,1} > 0$).

TABLE 1, Estimated parameters for the three state MRS model using Vasicek dynamics together with Gamma spikes for daily price in the Nord Pool System. Significant parameters are emphasized in bold. All time series are evaluated from 2006 Q1 until June 30th in 2013, except for the wind power that is estimated from 2009 Q1 to 2013.

Variable	$\beta_{BS,1}$	$\beta_{BD,1}$	$\beta_{SB,1}$	$\beta_{DB,1}$
Consumption	**28.14**	**-17.54**	**-16.34**	**7.71**
Production	**31.23**	**-16.75**	**-33.61**	4.42
Reserve Margin	1.11	0.08	0.45	-1.48
Wind power	1.66	1.20	**5.37**	-4.43
Variable	$\beta_{BS,0}$	$\beta_{BD,0}$	$\beta_{SB,0}$	$\beta_{DB,0}$
Consumption	**-27.84**	**6.53**	**13.27**	**-6.02**
Production	**-30.96**	**-5.87**	**29.80**	**-4.31**
Reserve Margin	**-4.45**	**-4.30**	**-1.79**	**-1.63**
Wind power	**-4.58**	**-4.52**	**-3.96**	**-0.48**

Moving on to hourly data presents a slightly different story, see Table 2, where we see all parameters that were significant when using daily observations still are significant using hourly observations. However, we also see that the reserve margin is an important variable, with all parameters being significant. Wind power is also important, with all relevant variables being significant, but we also note that the $\beta_{BS,1}$ parameter changed sign. Please note that the parameters are slightly different as the normalization of the external variable changed slightly, cf. Eq. (2).

TABLE 2, ESTIMATED PARAMETERS IN THE TRANSITION MATRIX FOR THE THREE-STATE MRS MODELS USING VASICEK DYNAMICS AND GAMMA SPIKES FOR HOURLY OBSERVED PRICES IN THE NORD POOL SYSTEM. SIGNIFICANT COEFFICIENTS ARE EMPHASIZED IN BOLD. ALL TIME SERIES ARE EVALUATED FROM 2006 Q1 UNTIL JUNE 30TH IN 2013, EXCEPT FOR THE WIND POWER THAT IS ESTIMATED FROM 2009 Q1 TO 2013.

Variable	$\beta_{BS,1}$	$\beta_{BD,1}$	$\beta_{SB,1}$	$\beta_{DB,1}$
Consumption	**34.12**	**-10.81**	**-3.64**	**2.87**
Production	**31.72**	**-12.14**	**-6.35**	**3.86**
Reserve Margin	**-4.17**	**0.81**	**0.81**	**-2.40**
Wind power	**-0.95**	**3.83**	**-9.78**	**-0.68**
Variable	$\beta_{BS,0}$	$\beta_{BD,0}$	$\beta_{SB,0}$	$\beta_{DB,0}$
Consumption	**-32.22**	**2.07**	**2.00**	**-3.62**
Production	**-30.99**	**2.64**	**4.30**	**-3.62**
Reserve Margin	**-12.93**	**-9.42**	**-0.19**	**-1.53**
Wind power	**-11.27**	**-10.40**	**-0.86**	**-2.10**

Models are often compared in terms of AIC or BIC, see [23]. This is equivalent to comparing the log-likelihood of the models in this paper (as all models have the same number of parameters), $log(L) = \sum log\ p(x_n|x_{\{1:n-1\}})$ where the transition probability $p(x_n|x_{\{1:n-1\}})$ is derived from the model. We find that the consumption still is the best explanatory variable by quite a margin, see Table 3. The estimated parameters also have the current signs, in the sense that their effects are what we expected them to be.

The reason why the consumption outperforms the reserve margin is not clear to us, but it could be that the reserve margin can be computed in several ways, as for example transmission capacity may be included in the definition. The numbers we used may therefore not represent the actual, controllable reserve margin well enough, cf. [8].

TABLE 3, LOG-LIKELIHOOD WHEN USING THE CONSUMPTION, PRODUCTION, RESERVE MARGIN AND WIND POWER AS EXPLANATORY VARIABLES, EVALUATED ON DATA FROM 2009-10-01 TO 2013.

Explanatory variable	$\log(L)$
Consumption	52 225
Production	51 381
Reserve margin	49 100
Wind power	49 087

IV. SIMULATION STUDY

We consider two winter weeks during the winter of 2015 that are not part of the data in Section III. First, we study the electricity spot price and consumption for the Nord Pool system between January 10th (which is a Saturday) until January 18th (which is the following Sunday). These data are presented in Fig. 2 (data can be downloaded from Nord Pool, http://www.nordpoolspot.com/historical-market-data/). It can be seen that the price increases somewhat during the working days (12/1-17/1) and peak hours, but also reverts to the normal price levels during off peak hours. The price peaks coincide fairly well with the consumption peaks.

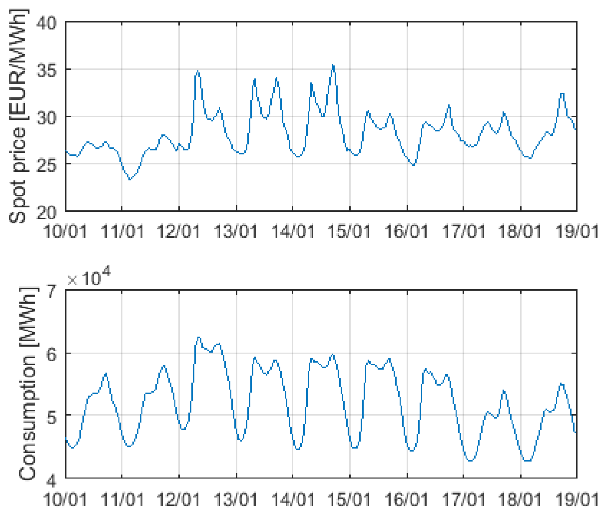

Fig 2. Electricity spot price (top) and consumption (bottom) for the Nord Pool system between January 10th (Saturday) and January 18th (Sunday) 2015.

It is well known that consumption is comparably easy to predict, see e.g. [24, 20] for an overview of methods. We can therefore forecast the consumption in order to assess the risks for spikes in the upcoming week. We have approximated the forecast by the actual consumption in the top panel in Fig. 3. The consumption is then used to forecast the probability for spikes by iterating the latent Markov chain governing the regime from 16/1 and onwards (middle panel). The increase in consumption from Monday to Friday translates into large spike probabilities. We also note that the persistence of the spike regime largely is determined by the consumption. These probabilities can be compared to the actual electricity spot prices during the same period (lower panel). We find that the model is able to predict the spikes, using only the consumption.

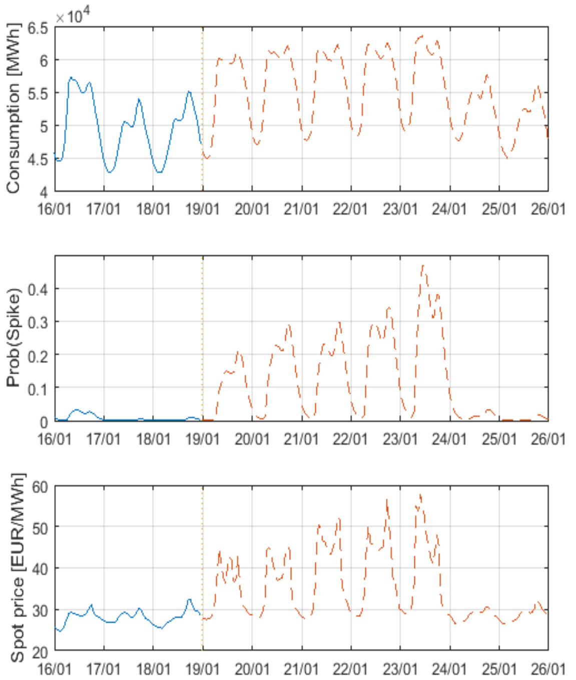

Fig. 3 Known data up until Sunday 18th January (solid line) and "Predicted" (dashed line) consumption for the upcoming week (top panel), the resulting spike probabilities (middle panel) computed from the consumption and actual electricity spot price (bottom panel) for the second week in January, 2015.

V. Conclusion

This paper presents an extension of the time invariant Independent Spike Model introduced in [7] by considering hourly observations rather than daily observations.

We found that the model can swiftly move from the base regime to the spike regime during peak hours only to revert later during the evening. It is now the external variable (typically consumption) that determines the persistence of the extreme prices. This is in stark contrast to the standard time homogeneous model where the persistent is constant (with exponentially distributed durations).

We also found support for the hypothesis that wind power and the reserve margin influences the probability for spikes and drops, but the likelihood based information criteria clearly ranked the consumption as the most important variable.

The model was used to demonstrate how consumption forecasts can be used to forecast spikes (including reversion to the base regime during off-peak hours). This could a very useful proxy for the stability in the physical power system!

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