Stress-dependent permeability of two-solid constituent model of gas shales

页岩气两固体成分模型的应力相关渗透率

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Abstract - **Because gas shales consist of organic and inorganic solid components, the stress and pore pressure dependence of permeability of shale gas rock is considered in the model of porous medium with two solid constituents. It is done in the framework of a generalized model of composite sphere phases each of them being gas/fluid saturated. It is shown that the total mean stress in each of the phases is different from the confining stress even for a homogeneous state. This leads to a more complicated dependence of permeability rather than that of Terzaghi effective stress, i.e. confining stress minus pore pressure as in the case of a one-solid constituent porous medium. Additional dependence of permeability on pore pressure is captured by considering Knudsen and slip flow contributions. Because depletion-induced variation of pore pressure leads to the variation of eigen-strain, the resultant strain and stress exerted at the reservoir by surrounding country rock is found by using Eshelby-type inclusion model. The depletion-induced evolution of reservoir permeability is expressed as a function of pore pressure. The model provides recommendation for maximum drawdown which still allows preventing collapse of porosity and permeability of softer organic constituent.**

Keywords – **gas shales, effective stress, permeability**

I. INTRODUCTION

Due to the fact the shale gas rock is constituted by at least two main solid components with sufficiently different mechanic properties: organic matter and inorganic one, we would like to consider how this feature affects dependence of effective medium permeability on stress and pore pressure. As usual we assume [1] that permeability, k, is some function of porosity, ϕ: $k(\phi)$. In conventional theory of poro-elasticity variation of porosity is a function of Terzaghi effective stress [2]:

$$\Delta\phi = (\alpha - \phi_0) \cdot \frac{\Delta p - \Delta\sigma_v}{K_b} \qquad (1)$$

where p is pore pressure, α is Biot's constant, $\Delta\sigma_v$ is increment of total mean stress (positive in compression), ϕ_0 is initial porosity, and K_b is drained bulk modulus of porous medium. The average stress, $\Delta\sigma_v$, in the eq. 1 relates to the stress in the part of rock, the porosity is defined for. If the rock has only one solid porous constituent, then the average stress is unique. Situation might be different if there are two or more solid constituents in porous medium. The local variations of stress in the different solid components are the same as global one only in some special cases [3]. Stress dependence of permeability is particularly important for the case of low porosity because stress increase may effectively close some flow paths. In that sense the system is close to percolation threshold. Due to small pore size, which is comparable with the mean free path of molecular motion, matrix permeability should also include Knudsen and slip flow components as well as conventional Poiseuille one [1]. It is important to know variation of reservoir permeability as a function of pressure as it changes as a result of depletion and compaction. It depends on interaction between reservoir and surrounding country-rock. For conventional reservoirs, it was described by Eshelby-type inclusion approach [4, 5] we would like to extend this approach to gas shale rock.

II. STRESS-DEPENDENT POROSITY IN PO-ROUS MEDIUM WITH TWO SOLID COMPONENTS

Gas shale rock has at least two solid constituents that are shale (with some carbonate content as well) and organic matter, i.e. kerogen. Unfortunately, at this point it is difficult to conclude from rock characterization data (SEM etc) which phase is continuous (connected) and which phase can be considered as an inclusion. The case when both phases (organic matter and shale) are continuous cannot be ruled out.

We generalize a simple model of composite material, specifically a "composite sphere model" (see [6] and reference therein) which was originally developed for non-porous composite media.

The structure of this generalized model can be described as a fractal one because the composite spherical arrangements of two phases of different gradation are assembled in the filling configuration while having the same proportion of two phases in them (denoted "*m*" and "*i*") as it is shown at the Fig. 1. Each phase is a porous material with its own porosity and mechanical properties. In our model both phases, *m* and *i*, are saturated with pore fluid. The inner sphere is constituted by porous phase "*i*" with radius denoted as *a*.

The phase "*m*" fills the space between the boundary of inner sphere and the boundary of outer sphere, denoted as *b*. It is assumed that distribution of composite spheres with respect to their total radius *b* is random but in each sphere the ratio of *a/b* is the same. This means that volumetric fraction of phase "i" in the whole rock is equal to $c = (a/b)^3$. We consider that mechanical properties are different in *m* and *i* phases including Biot's coefficients. It should be noted that we consider here a homogenous state of equilibrium, so the pore pressure is constant and is the same in both phases.
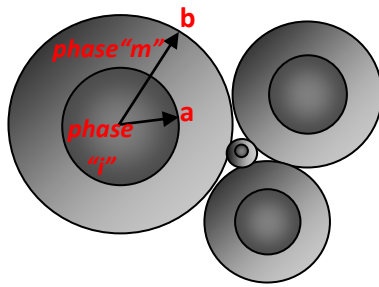

Figure 1. Composite porous sphere 3D model (schematic)

Using results of this model we can find stress in all constituents separately. For the sake of brevity we omit intermediate calculation and give formulas for mean total stress in both phases. A more detail formulation of the model is described in a separate publication [7].

Average total stress in *m*-phase

$$\sigma_{m,v} = \frac{p_c}{1+cf} - \frac{p}{1+cf}\left[\alpha_m + c\frac{4G_m(\alpha_i - \alpha_m)}{3K_i + 4G_m}\right] + \alpha_m p \quad (2)$$

and mean total stress in phase *i*:

$$\sigma_{i,v} = p_c\frac{1+f}{1+cf}$$
$$-p\frac{1+f}{1+cf}\left[\alpha_m + \frac{(3K_m + 4G_m c)(\alpha_i - \alpha_m)}{3K_m + 4G_m}\right] + \alpha_i p \quad (3)$$

where coefficient

$$f \equiv \frac{4G_m(K_i - K_m)}{K_m(3K_i + 4G_m)} \quad (4)$$

p_c is confining stress, p is pore pressure, c is volume fraction of i phase, K and G are drained bulk and shear moduli respectively, and α_i, α_m are Biot's coefficients in i and m phases respectively. The stresses in i- and m- phases are shown at Figures 2 and 3.
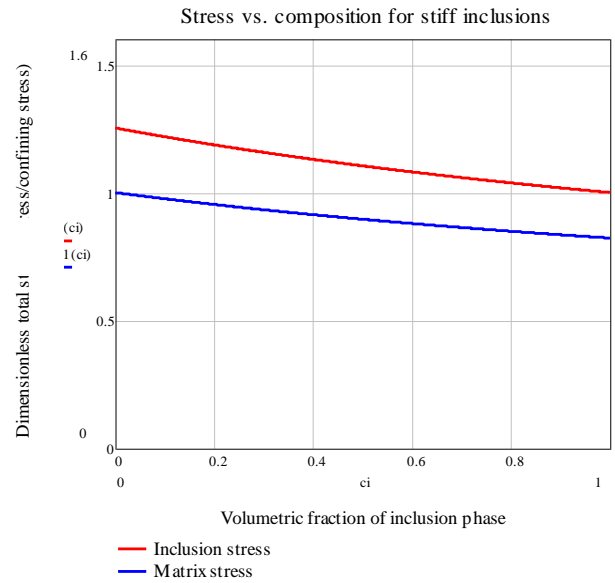

Figure 2. Total dimensionless stresses for matrix, $\sigma_{m,v}/p_c$ and inclusion, $\sigma_{i,v}/p_c$ vs. composition c in inclusion and matrix phases when inclusion is stiffer than matrix: $K_i=3K_m$.
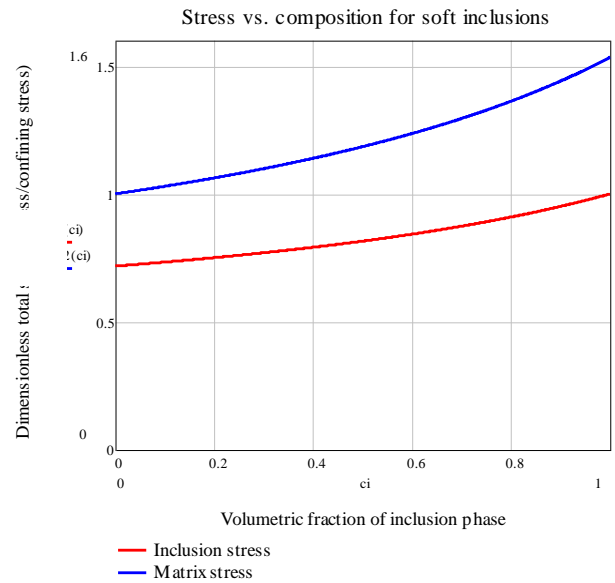

Figure 3. Total dimensionless stresses for matrix $\sigma_{m,v}/p_c$, and inclusion, $\sigma_{i,v}/p_c$, vs. composition c in inclusion and matrix phases when inclusion is softer than matrix: $K_i=K_m/3$.

The average stress in the rock is calculated as follows:

$$\sigma_v = c\sigma_i + (1-c)\sigma_m \quad (5)$$

and after substitution of eqs. (2) and (3) into (5), we obtain for total average stress:

$$\sigma_v = p_c \tag{6}$$

as it should be under exerted confining stress p_c. For effective average stress we obtain from:

$$\sigma'_v = p_c - \alpha p \tag{7}$$

and effective Biot's coefficient for two-solid-component porous medium reads:

$$\alpha = \frac{c\alpha_i \,(3K_m + 4G_m) + (1-c)\,\alpha_m \,(3K_i + 4G_m)}{c\,(3K_m + 4G_m) + (1-c)\,(3K_i + 4G_m)} \tag{8}$$

The expression for porosity (1) is valid for each porous phase separately, if using the mean stress for the given phase. After substitution of the expression for mean stress eq. (2) and (3) in the eq.(1) we obtain for stress dependent porosity:

$$\phi_m = \phi_{0m} - (\alpha_m - \phi_{0m}) \cdot \frac{\Delta p_c - \gamma_m \,\Delta p}{(1 + c f)\,K_m} \tag{9}$$

where coefficient

$$\gamma_m = 1 + c f\,(1 - \alpha_m) + c\,\frac{4G_m\,(\alpha_i - \alpha_m)}{3K_i + 4G_m} \tag{10}$$

As we can see coefficient γ_m can be potentially higher than 1 if the phase m is softer than phase i. For a single solid constituent porous medium, when $c = 0$, coefficient γ_m is equal to unity as it should be in conventional theory of poroelasticity for one-solid component porous medium.

Similarly for the i-phase porosity is expressed as follows:

$$\Delta\phi_i = -(\alpha_i - \phi_{0i}) \cdot \frac{1 + f}{(1 + c f)\,K_i}\,(\Delta p_c - \gamma_i \,\Delta p) \tag{11}$$

where coefficient

$$\gamma_i = (1 - \alpha_i) \cdot \frac{1 + c f}{1 + f} + \alpha_m + \frac{(3K_m + 4G_m c)\,(\alpha_i - \alpha_m)}{3K_m + 4G_m} \tag{12}$$

Again when the phase "i" occupies the whole volume, so that for $c = 1$, the coefficient $\gamma_i = 1$ as in conventional theory of poroelasticity. Numerical behavior of the coefficients γ_i and γ_m are shown at Figures 4, 5.

It should be noted that our model should not be mixed up with Zoback-Byerlee model [8] which explored a stiff and soft materials arrangement around each pore, whereas we consider two macroscopic porous phases of different stiffness.

Different effective stress rule for porosity arises due to difference in mean stresses for the phases m and i from the average mean stress in the whole porous medium as it is shown at Figs. 2 and 3 for the cases of soft and stiff inclusions.
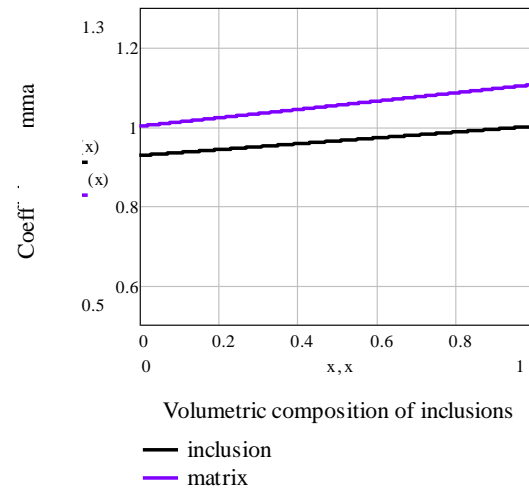

Figure 4. Pore pressure coefficients γ_i and γ_m given by eqs. (10) and (12) when inclusion is stiffer than matrix: $K_i = 3K_m$.
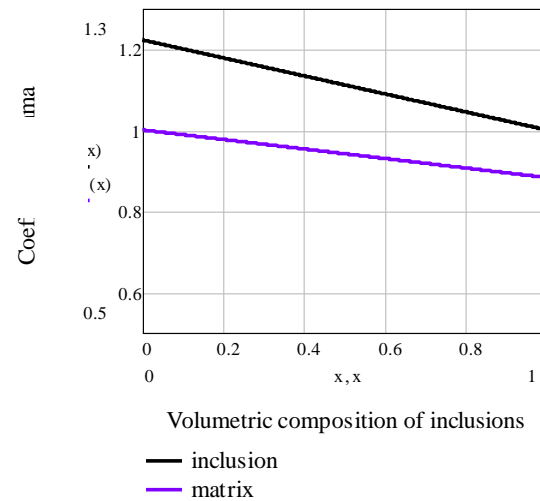

Figure 5. Pore pressure coefficients γ_i and γ_m given by eqs. (10) and (12) when inclusion is softer than matrix: $K_i = K_m /3$

The Biot effective stress rule for strain and Terzaghi effective stress rule for porosity still holds true but separately in each of the constituent phases.

As we can see from Figs. 2 and 3 our model gives correct crossover transition to the limit cases of one-component porous media of $c=0$ and $c=1$ when total stress must be equal to confining stress p_c (or in dimensionless units equal to 1).

III. PERMEABILITY OF COMPOSITE MEDIUM

There were numerous efforts in the past to derive effective properties of composite media. Effective permeability can be estimated approximately [6] as follows:

$$k_{eff} = k_m \left[1 + \frac{3c}{(1 - c) + 3k_m /(k_i - k_m)} \right] \tag{13}$$

Because in general permeability is a function of porosity [1]: $k = k(\phi)$, after using explicit expression for porosity in inclusion and matrix (9), (11) we obtain that permeability of composite medium is a function of the following variables:

$$k_{eff} = k\left(p_c - \gamma_m p, p_c - \gamma_i p, p\right) \qquad (14)$$

If permeability of *i*-phase (inclusion) is negligible, then from eq. (13) effective permeability:

$$k_{eff} \approx k_m \frac{2 \ (1-c)}{2+c} \qquad (15)$$

which means that effective permeability reads:

$$k_{eff} \approx k \ \ (p_c - \gamma_m p, p) \qquad (16)$$

For non-adsorbing gas saturation, the separate dependence of permeability on pore pressure (the second variable in the eq. (16) is only due to Knudsen and slip flow. It means that for high pressure when these effects are not important, and the stress dependence of permeability reduces to one variable, p_c-γp.

If experimental measurements of permeability of gas shales can be fitted better by some linear combination of confining stress and pore pressure: p_c-χp with $\chi \neq 1$ [9, 10] it does not mean that Terzaghi effective stress rule for porosity is not valid but could be an indication that the rock contains several solid constituents with different elastic properties of the components. It is the case for gas shales where kerogen constituent is significantly softer (-3-4 times) compared to inorganic constituent (e.g. calcite).

Finding an adequate description of permeability in shale gas is a challenging problem because of complexity of the rock which contains both organic and non-organic components. It is also important to capture the following features of gas shale systems:

1. A model of Klinkenberg effect: due to small pore size, which is comparable with the mean free path of molecular motion, matrix permeability should include Knudsen and slip flow components as well as conventional Poiseuille one. In order to address this feature we have implemented Scott-Dullien model [11] as well as Ottani-Wakao-Smith model [12]. Both of the models are able to provide crossover from viscous Poiseuille flow in pores to Knudsen flow with molecular streaming for small pores.
2. Because gas methane can be adsorbed and desorbed by solid constituents (primarily by organic phase) the sorption-desorption induced strain and associated with it stress should be taken into account. We have implemented the open-system geomechanics approach as that in a system with variable solid mass (see [7] and references therein);
3. Estimation of reservoir permeability vs. pore pressure only based on the model results and Eshelby inclusion approach which allows to exclude stress.
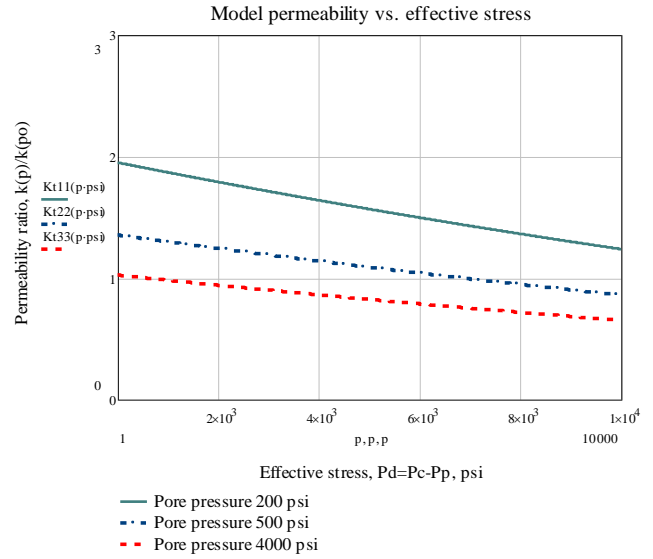   These details can be found in our extended paper [7] and briefly presented at the Figs. 6 and 7.

Figure 6. Model permeability as a function of pore pressure; effective Terzaghi stress, p_d, is equal to confining stress minus pore pressure: $p_d = p_c - p$.
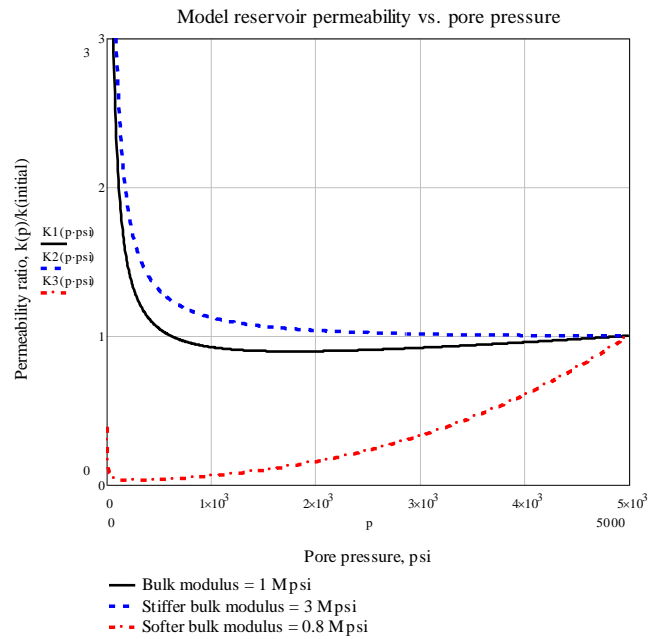

Figure 7. Permeability vs. pressure for different values of bulk modulus K_m.

IV. CONCLUSION

- Stress and pore pressure dependence of permeability is needed for:
  - Adequate drawdown management – not to collapse porosity and permeability near the well during well ramp-up;
  - Estimation of permeability evolution during depletion.

- Based on these reasons a new permeability model of shale gas with two solid porous constituents is derived; the model captures dependence on stress and pore pressure. Stress and pore pressure dependence brings three additional dimension-less parameters, *f*, γ_m, , γ_i, eqs. (4) - (12) which are explicitly expressed through poroelastic mechanical properties of both phases.
- The model predicts existence of compaction-induced minimum of permeability as a function of pore pressure. There is no minimum of permeability with pore pressure decrease in sufficiently stiff formations.
- Dependence of permeability vs. stress and pore pressure, given by Figures 6, 7, is in qualitative agreement with experimental results [9, 10].
- More measurement of permeability as a function of stress and pore pressure are needed to have necessary input parameters and verify the model.

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