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Simulation of multi–stage hydro–fracture development by the SIE method

通过 SIE 方法模拟多级水力裂缝的发展

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Abstract – **This study deals with 2D simulations of curvilinear multi–stage hydro-fracture growth in a reservoir. The numerical model employs the method of complex singular integral equations, the SIE method. The crack path is found by applying the criterion of maximum tensile stress at the fracture tip. The study investigates the fracture development for different initial spacings between the fractures, in-situ stresses and the values of fluid pressure that can be different for different cracks.**

Keywords – **Hydraulic fractures, Crack growth, Complex singular integral equation**

I. INTRODUCTION

Collective fracture growth driven by internal fluid can be observed in nature and during technological processes. Examples include magma penetration into the crust, which causes dike and sill formation or multistage hydraulic fracturing. Nowadays the latter becomes a widely used procedure of enhancing well productivity in low-permeability reservoirs (Cippola et al. 2009), which requires proper design to provide economic effectiveness of this treatment. Conventional design does not take into account the stress field redistribution induced by the fracture interaction, which affects the crack trajectories and deviate them from the straight path. This can lead to a complex geometry of the fracture system especially in the case when the fractures are placed close to each other. Any fracture developed under the action of internal pressure caused by fluid penetration is further referred to as hydraulic fracture (HF). In this study multi–stage HF growth is modeled. It means that the fracture system is build up subsequently (the next HF starts to grow when the previous one stops). The numerical model is based on the method of singular integral equations (SIE), which provides effective computation of the fracture characteristics used further on for calculations of the fracture path.

II. PROBLEM FORMULATION AND METHOD OF SOLVING

The physical model is based on the mechanics of brittle fracture. Our aim is to calculate quasi-static fracture trajectories; therefore the process of fracture propagation is modeled by a sequence of the stationary states (steps). At each step of fracture growth the plane elastic problem is solved for which we employ the method of complex SIE in the form developed by Savruk (1981). The system of SIE is solved numerically by the method of mechanical quadratures based on the Gauss-Chebyshev quadrature formula (see Savruk, 1981). It is assumed that either the loads or the crack opening displacement (or its derivative) are known on each fracture. On the fractures where the loads are known the crack opening displacement (or its derivative) is the sought function, which is used for calculating the stress intensity factors (SIFs) for the current configuration of the fracture system. We assume that the fracture continue to propagate if the mode I stress intensity factor K_I is greater than the fracture toughness K_{IC}. For crack path simulations we apply the criterion of maximum tensile stress at the crack tip. The fracture closure is checked at every step to ensure that the crack surfaces do not enter into contact over any part of the crack. The process ends when a certain fracture length is reached or if the crack surfaces start to contact. In the latter case the fracture growth terminates prematurely (certain length is not reached), because the system of SIE does not describe this case and needs to be modified.

III. MULTI-STAGE HF SIMULATION: TWO APPROACHES

To simulate the multi-stage (subsequent) growth we assume that every new fracture appears and starts to grow only after the previous one stops when it achieved certain length, i.e. during the n^{th} stage the n^{th} fracture is the growing one and the length of the (n-1) previously developed cracks are kept equal to an assigned value (same for all fractures). All initial fractures are assumed to be straight and oriented coaxial to the major compressive in-situ stress T_2 (T_1 is the minor compressive in-situ stress). We further assume that these are normalized by the value of pressure in the firstly growing crack (the straight crack shown on the left in all subsequent figures). Therefore, the following constraint is imposed $0<T_1<1$, if $T_1 \geq 1$ the hydrofracture cannot grow.

To model multi-stage HF growth we consider two approaches. In the first one we take into account the mutual interaction between the fractures. Physically it means that not only the previously grown crack affects the growing one (direct influence) but also the growing crack affects all previous cracks. In mathematical formulation we impose the in-situ stresses and the fluid pressure in each fracture. While the n^{th} fracture is growing the crack opening displacement (and consequently the SIFs) of the previous n-1 cracks can change their values because of the influence of the growing crack due to increase of its length.

In the second approach we do not take into account the mutual interaction of all cracks. It implies that there is no back influence of the growing HF on the previous ones, so only the previously made cracks affect the growing one. It leads to a different mathematical formulation of the problem. Namely, on the previous (n-1) fracture the crack opening displacements obtained at the previous stages are fixed whereas on the currently growing fracture we impose the load: fluid pressure, in-situ stresses and the stresses caused by the previously grown cracks. As the result it is necessary to solve only one complex SIE instead of n generated in the first approach. Thus, from the computational side the second approach requires less computer resources.

Both approaches are important because they represent two limiting cases. Therefore, the real crack path should lie somewhere between the two trajectories simulated in accordance with these approaches. This is why all of the figures presented in this study show two trajectories obtained by both approaches (the first approach is presented by dashed lines and the second approach by continuous lines).

There are several factors that affect the crack path and in this study we examine some of them.

IV. Trajectory Dependence on the Distance

It is evident that for larger distances between the fractures their interaction is less pronounced. Fig. 1 shows the results of simulation for different relative spacing between the fractures $d/2a$, where d is the distance between the nearest crack centers and $2a$ is the final fracture length. In all cases presented in this paper the units for horizontal and vertical axes are dimensionless length and we specify the final fracture half-length a equal to ten units. Fluid pressure is assumed to be uniform and equal to one unit. The values of the in-situ stresses are equal and do not exceed the fluid pressure. Their value in this case does not affect the fracture trajectories. The mode I stress intensity factor K_I is assumed to be greater than the fracture toughness K_{IC}. It the latter is negligible, then the fracture grows occur for $K_I > 0$.

The fracture trajectory is modeled by a polygonal line with equal sides; same for all fractures. Two sides are added symmetrically to the crack ends at every step of fracture growth to the previous configuration. The crack system is formed by adding new fractures from left to right as shown in all figures below.

Simulation shows that the decrease of spacing pushes the cracks to deviate more and more from the straight path as demonstrated in Fig 1a-d. Eventually it leads to fracture closure that initiate termination of further calculations. It should be noted that closure occurs for the crack on the right in Fig. 2.

Fig. 3-5 present the range of simulation of multi-stage HF for in-situ stresses T_1 =0.4 and T_2 =0.8 and the fluid pressure of one unit for different dimensionless spacings $d/2a$. At each step of the fracture growth its closure is controlled. The cracks grow until the assigned length. The trajectories obtained by both approaches are closed to each other in most cases.

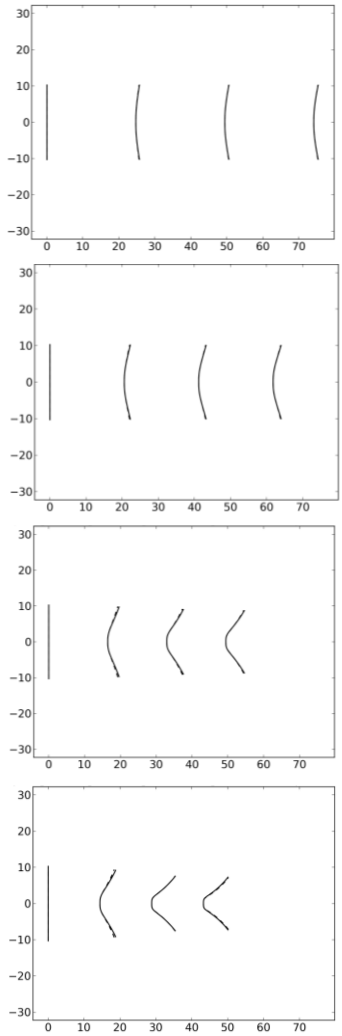

Fig.1, Multi-stage HF driven by fluid pressure of one unit and equal in-situ stresses $T_1 = T_2$ =0.6 for: a) $d/2a$=1.2 b) $d/2a$=1.0 c) $d/2a$=0.8 d) $d/2a$=0.7.

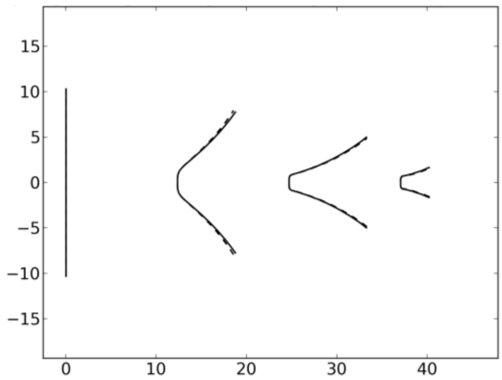

Fig.2, Fracture growth terminates prematurely. Multi-stage HF driven by one unit fluid pressure with equal in-situ stresses of $T_1=T_2=0.6$ for $d/2a=0.6$.

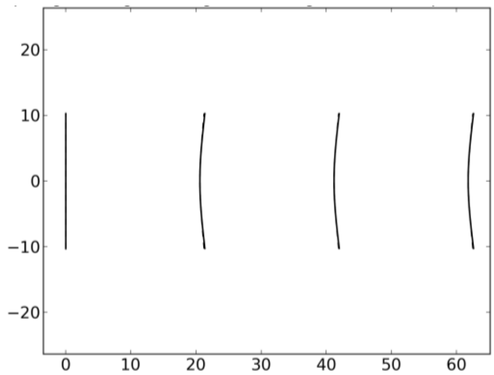

Fig.3, Multi-stage HF driven by one unit fluid pressure; $T_1=0.4$ and $T_2=0.8$ for $d/2a=1.0$.

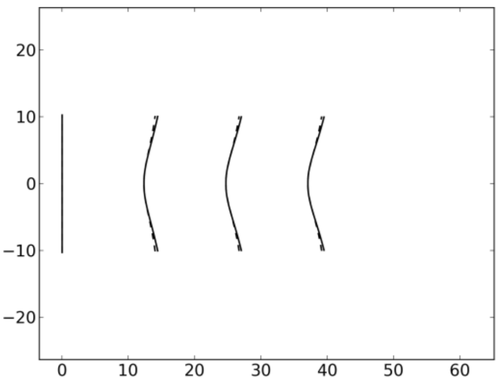

Fig.4, Multi-stage HF driven by one unit fluid pressure; $T_1=0.4$ and $T_2=0.8$ for $d/2a=0.6$.

The results of simulation show that the contrast of in-situ stresses makes the crack path to be less curvilinear, which can compensate its further bending due to decrease of relative spacing. Therefore it is possible to reduce the initial spacing between the multi-stage HF in reservoirs subjected to high contrast in-situ stresses (Fig. 5) as compared to the case of hydrostatic in-situ stresses (Fig. 2).

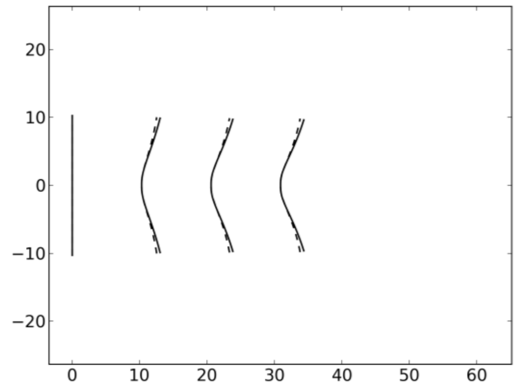

Fig.5, Multi-stage HF driven by one unit fluid pressure; $T_1=0.4$ and $T_2=0.8$ for $d/2a=0.5$.

V. THE INFLUENCE OF THE IN-SITU STRESSES

Let us introduce the in-situ stress ratio $k=T_2/T_1$, $k>1$ that characterizes the contrast of in-situ stresses. We have already conducted some simulation for $k=1$ (Fig. 1- 2) and $k=2$ (Fig. 3-5). The results of simulation for $k=4$ (these are not presented in the paper) is similar to the case k=2. Thus, it can be concluded that further increase of the in-situ stress ratio makes no significant effect on the curvature of the trajectories as presented in Fig. 5. Furthermore, usually in the real reservoir the value of k is between 1 and 2. Fig.6 and 7 show the results of stimulation for $k=1.1$ and $k=1.23$.

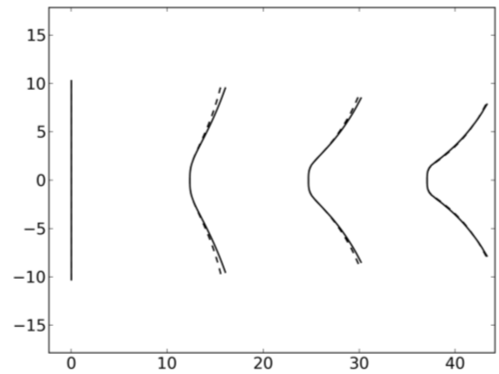

Fig.6, Multi-stage HF driven by fluid pressure of one unit; $T_1=0.727$ and $T_2=0.8$ (k=1.1) and $d/2a=0.6$.

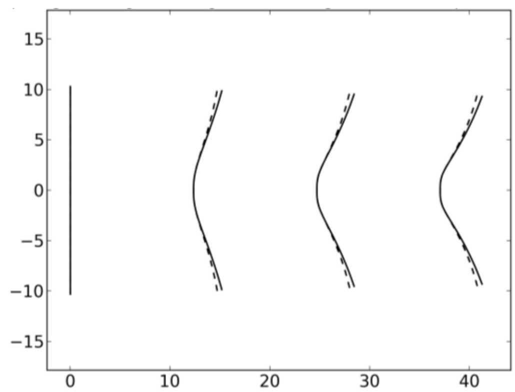

Fig.7, Multi-stage HF driven by fluid pressure of one unit; $T_1=0.65$ and $T_2=0.8$ (k=1.23) and $d/2a=0.6$.

The results of simulation (see Fig. 2,4,6,7 made for the same spacing $d/2a$=0.6) show that the increase of the in-situ stress ratio leads to straightening of the HF trajectories. Comparing the trajectories in Fig. 2 and Fig. 6 one can notice that even a small stress contrast makes the fracture path straighter. Therefore, the situation presented in Fig. 2 is unlikely to happen in practice.

VI. The Influence of the Different Pressures

In all configurations presented above both approaches (with and without mutual integration) produce quite similar trajectories (dashed and continuous lines are hardly distinguishable). However, they can be essentially different as shown further on.

Fig. 8-13 present the results of simulations with different fluid pressures in different HF. In-situ stresses are T_1=0.4 and T_2=0.8 (k=2) for the cases shown in Fig. 8-10; Fig. 11 addresses the case T_1=T_2=0; Fig. 12 the case T_1=T_2=0.6 (k=1); and Fig. 13 the case T_1=0.65, T_2=0.8 (k=1.23). The relative spacing in all these cases is $d/2a$=0.5. Fluid pressures are assumed to be uniform, their values are given by the formula $p_{hf}=q^{s-1}$, where s is the number of the stage (the number of fractures counted from left to right) and q>1 is a parameter that vary from figure to figure.

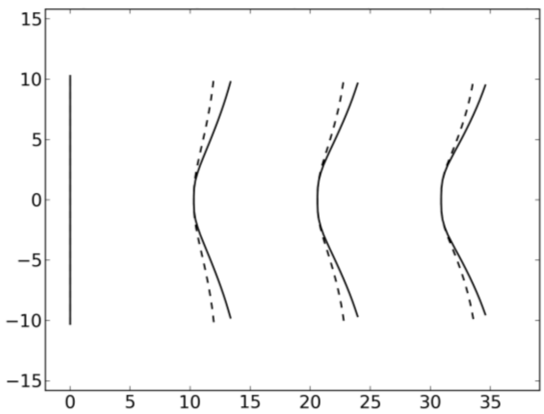

Fig.8, Multi-stage HF driven by fluid pressure phf=qs-1, q=1.1; T1=0.4, T2=0.8 and d/2a=0.5.

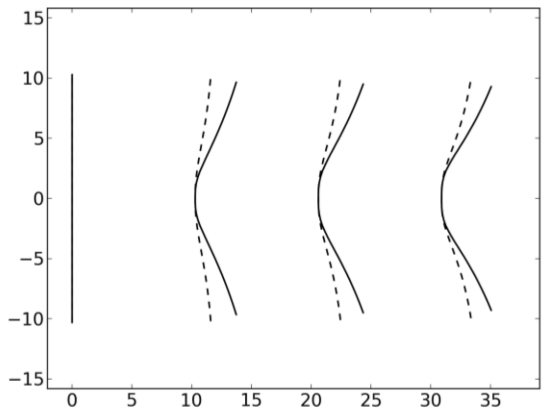

Fig.9, Multi-stage HF driven by fluid pressure $p_{hf}=q^{s-1}$, q=1.2; T_1=0.4, T_2 =0.8 and $d/2a$=0.5.

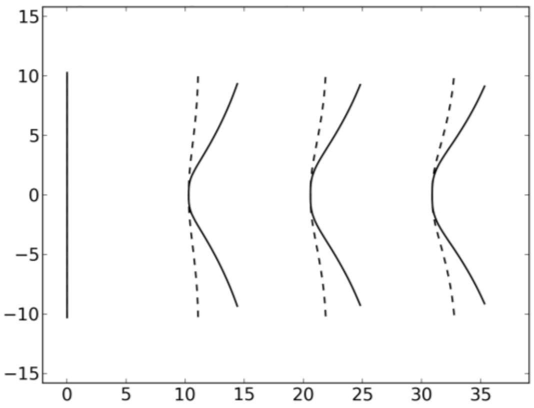

Fig.10, Multi-stage HF driven by fluid pressure $p_{hf}=q^{s-1}$, q=1.4; T_1=0.4, T_2=0.8 and $d/2a$=0.5.

Comparing Fig. 5 and Fig. 8-10 one can notice that the trajectories calculated by the different approaches are essentially different when the pressure rises (by increasing q). They not only deviate from each other, they also show a qualitatively different response on pressure rise. Namely, the crack paths have tendency to straighten if the crack interaction is taken into account. If it is not, they tend to bend more and deviate from the straight line.

Fig. 11-12 depict crack behavior in the case of equal in-situ stresses. They are zero in the configuration presented in Fig. 11 and equal to 0.6 in Fig. 12.

Fig. 13 represents the crack path for in-situ stress ratio k=1.23 (T_1=0.65, T_2=0.8).

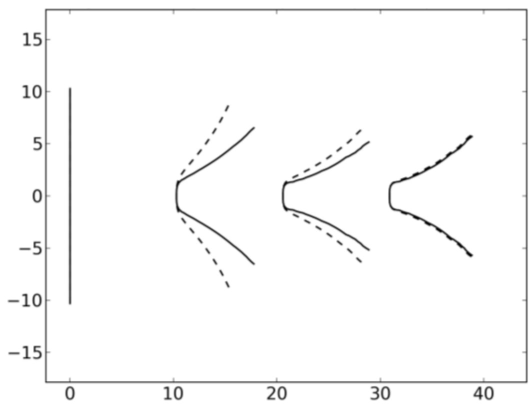

Fig.11, Multi-stage HF driven by fluid pressure $p_{hf}=q^{s-1}$, q=1.2; T_1=T_2=0 and $d/2a$=0.5.

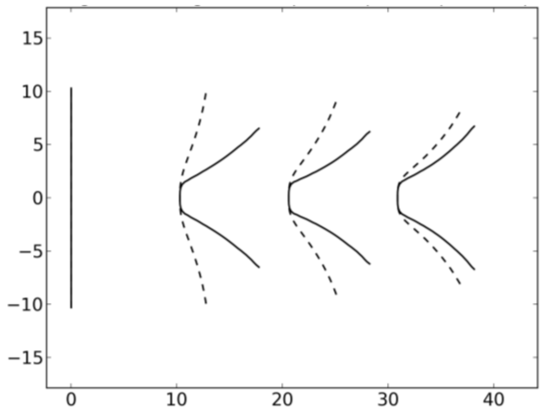

Fig.12, Multi-stage HF driven by fluid pressure $p_{hf}=q^{s-1}$, q=1.2; $T_1=T_2$=0.6 and $d/2a$=0.5.

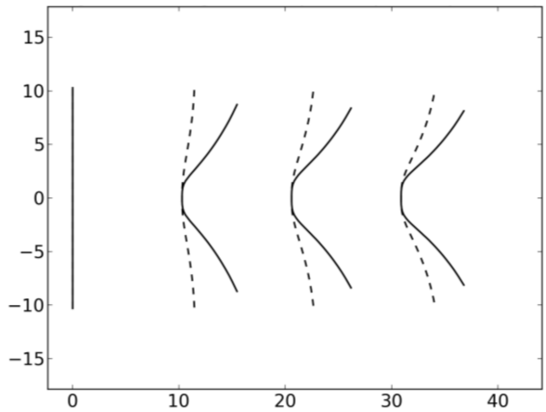

Fig.13, Multi-stage HF driven by fluid pressure $p_{hf}=q^{s-1}$, q=1.2; T_1=0.65 and T_2=0.8 (k=1.23) and $d/2a$=0.5.

The results of simulations presented in Fig.8-13 show that in the case of different fluid pressures the values of the in-situ stresses play a significant role, even if k=1 (especially in simulations without taking into account the mutual interaction of all cracks). It has been found that even small increase of q leads to apparently different trajectories obtained by two approaches.

VII. Shielding Effect and its Compensation

Let us discuss why the different values of fluid pressure (q>1) strongly affect the crack path. The first reason has already been mentioned, the fractures tend to deviate less from the straight line when the in-situ stress ratio is close to unity (compare the trajectories in Fig.2 and Fig 12). The second reason is, so called, shielding effect. The latter means the decrease of K_I values of the currently growing crack due to influence of the previously grown cracks. Consider the multi-stage HF driven by fluid pressure for in-situ stresses of T_1=0.4 and T_2=0.8 and relevant spacing $d/2a$=0.5. Fluid pressure is equal to one unit in every fracture (equally loaded cracks). The calculated fracture trajectories are presented in Fig.5. The values of the mode I stress intensity factor K_I for the growing crack are summarized in Table I depending on the fracture length, 2a, shown in the first column.

TABLE 1, Mode I SIFs for growing fracture for equally loaded crack

Length	1 stage	2 stage	3 stage	4 stage
0.6	0.58	0.21	0.17	0.16
4.6	1.61	0.65	0.56	0.54
11.0	2.49	1.38	1.24	1.20
19.8	3.35	2.54	2.39	2.34

One can notice significant decrease of K_I for the 2, 3, 4 fractures during their growth compare to K_I for the first stage respectively. In this case the shielding effect can make K_I to be smaller than the fracture toughness (or even negative), which stops fracture growth. Furthermore, if K_I is small then the crack opening is small too. Insufficient crack opening may impede the transfer of proppant along the fracture and eventually decrease the overall productivity of the HF system. Increase of fluid pressure in each following HF is capable to compensate the shielding effect. Table 2 shows the values of K_I for the growing cracks during the simulation for the same configuration except of the values of the fluid pressure specified as $p_{hf}=(1.1)^{s-1}$, where s is the number of the growing HF. The trajectories of this simulation are presented in Fig.8.

Comparing Table 1 and 2 one can notice that the increase of 10% in fluid pressure of the next fracture (q=1.1) provides considerable compensation of the shielding effect.

TABLE 2, Mode I SIFs for the growing fracture for differently loaded cracks

Length	1 stage	2 stage	3 stage	4 stage
0.6	0.58	0.30	0.31	0.34
4.6	1.61	0.92	0.96	1.07
11.0	2.49	1.81	1.96	2.20
19.8	3.35	3.20	3.55	3.97

VIII. Conclusion

In this study we have simulated the fracture trajectories during the mutli-stage HF by using the SIE method.

We have analyzed the influence of different factors on the crack trajectories and the stress intensity factors. It was shown the crack paths deviate from the straight line more and more as soon as the spacing decreases. The increase of in-situ stresses ratio (parameter k) forces the crack paths to straighten. Also it was found that the increase of fluid pressure of the next fracture over the previous one is capable of compensating the shielding effect, making the trajectories straighter (especially in the case of hydrostatic in-situ stress field), and reduce the probability of their closure.

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