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Research on the local and total stability criterion of high arch dam

高拱坝局部和整体稳定性评判准则研究

Guojian Shao（邵国建）*, Jiashou Zhuo（卓家寿）

Department of Engineering Mechanics, Hohai University, Nanjing 210098,China

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Abstract - **It is essential for the design of high arch dams to research problems of ultimate bearing capacity and total stability of the high arch dams. On the basis of evaluation on current analysis methods of the stability in the arch dam abutment and the total arch dam, their deficiencies are pointed out. With the application of the friction theory and the vector geometry, the formula of anti-slide stability safety coefficient is presented based on the non-linear FEM analysis. In response to the stability problem of high arch dam abutment without conspicuous slide faces, the local and total stability of the high arch dams are researched using the non-linear FEM analysis. Based on quantitative disturbing energy criterion and static criterion, the quantitative standards of the latent slide face and the most dangerous slide direction and the minimum stability safety coefficient are proposed and established. The stability criterion system is perfected for the local and the total stability of the high arch dams. The mechanical foundations are laid for quantitative stability criterion on the local and the total stability of high arch dams without conspicuous slide faces. The stability computation of a high dam abutment with the height of 305m is given. According to the disturbing energy value and its isograph, the latent side faces are determined. Estimating rockmass stability by the disturbing energy method and the static method, the numerical results indicate the rationality and the feasibility of the presented method.**

Keywords – **high arch dam, dam abutment stability, static criterion, disturbing energy criterion, stability safety coefficient.**

摘要 – 高拱坝的极限承载力和整体稳定性是高拱坝设计中的关键问题，在总结和评价现有拱坝坝肩和坝基以及整体稳定性分析方法的基础上，指出其中的不足之处，针对没有明显滑动面的拱坝坝肩岩体稳定性问题，采用三维非线性有限元理论与分析方法围绕高拱坝局部和整体稳定性问题进行针对性的研究。基于可量化的干扰能量准则和静力准则，建立了确定潜在滑动面、最危险滑向和最小安全系数的失稳警戒指标的量化标准，完善了拱坝局部与整体稳定性的评判体系，从而为解决没有明显滑动面的拱坝局部和整体稳定性评判量化指标这一难题奠定了力学基础。

关键词 – 高拱坝, 坝肩稳定, 静力准则, 干扰能量准则, 整体稳定安全度。

I. INTRODUCTION

An arch dam is the structure integrally bearing and retaining water, whose load is passed to the rock body in two banks by the construction base level. The arch dam balance is an entire equilibrium problem of dam body, rock mass and the interface between them. Arch dam stability is that keep dam body, rock mass and the interface in balance entirely under powerful water load and so on. Among dam body, rock mass and the interface, if one of them becomes unstable, the whole arch dam is in instability then. In order to keep the whole arch dam in stability, the dam body and rock mass and the interface have to maintain theirs equilibrium status severally. At the present time, evaluating arch dam local and total stability methods include rigid limited equilibrium method[1], geomechanical model test[2],over loading method based on nonlinear finite element, and reduce the material strength method[3]. Rigid limited equilibrium method obviously is limited in both conception in solving methods, based on quoting some assumptions and neglecting many factors, which lead to inexact computed results. Geomechanical model test has difficulties in finding some modeling materials which can be used to simulate dam body concrete and dam foundation completely according to similarity theory requests; in simulating high arch dams complex load mode and many nonlinear influences also.

If abutment's possible sliding mass is known in advance, anti-slide force and sliding force on sliding surface are obtained from non-linear FEM calculating results, and possible sliding mass's anti-slide stability safety coefficient can also be gained through projection. In present anti-slide stability safety coefficient computing formulae[4], synthesis of anti-slide force and sliding force on sliding surface often adopt simple algebraic addition and leading to considerable difference in the results. Besides, if possible sliding mass is not given in advance, abutment stability cannot be estimated. Although the reference [5] adopts the point safety coefficients to judge the stability, the point safety coefficients in essence is strength condition in classical mechanics and only reflects Mohr-Coulomb strength criterion.

Under the background above, after summarizing and appraising existing analysis methods of arch dam abutment and foundation and total stability, with the application of the friction theory, the vector geometry formula of anti-slide stability safety coefficient of three-dimension problem is presented based on the non-linear FEM analysis. According to the mechanics principle of engineering stability, the disturbing energy method of rockmass stability criterion is presented which is used to propose and establish the latent slide mass and the most slide direction and the corresponding stability safety coefficient. The judging system of the joint applied static method and disturbing energy method is promoted, which makes base for the local and total stability quantification judgment of the arch dams.

II. RIGID LIMITED EQUILIBRRIUM METHOD

At present, the rigid limited equilibrium method is the conventional design method in analysis of structural and slope stability at home and abroad, it has engendered relevant safety coefficient conception based on engineering experience, and also be brought into kinds of design specification. It has been acquainted, habitual and traditional to the largeness designers. The rigid limited equilibrium method assumes that the possibly gliding mass is a rigid body of no deformation and the force system effects on gliding mass only includes the normal force and the shearing force but no bending moment. The ratio of anti-slide force and sliding force is defined as stability safety coefficient of slide mass. There are a lot of methods, the main computing equation of stability safety coefficient is

$$K = \frac{\sum N_i f_i + \sum c_i A_i}{\sum T_i} \tag{1}$$

In Eq. (1), N_i, f_i, c_i, T_i, A_i indicate respectively effective normal force, friction factor, cohesive force, sliding force of the glide direction and the area of the sliding surface on the slide mass, the subscript i denotes numbered i region of the area of the sliding surface. After getting the N_i, T_i through the projection equilibrium method of slide mass force system, then substituting them into Eq. (1), we can get the value of stability safety coefficient.

The modal of rigid limited equilibrium method is shown in Fig. 1.

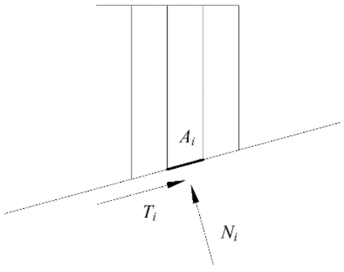

Fig.1 The modal of rigid limited equilibrium method

Because the rigid limited equilibrium method imports some assumptions and many factors neglected, the calculation results is rough comparatively and it can't reflect stress status and failure mechanism of rock mass structure or dam abutment accurately; and the stability safety coefficient we get is almost estimated value. But the limit equilibrium method can't resolve of the stability problem of complex slide mass which have strike fault or crevassed structure, particularly to the instability problem of arch dam abutment rock.

III. THE GEOMECHANICAL MODEL TEST

Three are three major ways in the model test of high arch dam total stability. They are over loading method and strength reserve method and synthetical approach. Over loading method assume the dam foundation rock mechanical parameters is unchanged and gradually increase the upstream water load until the foundation rupture instability, over loading multiple of water load is called overloading safety degree; Strength reserve method considers that the dam foundation rock itself has a certain strength reserve capacity and gradually lowers the design mechanical parameters of the rock until the foundation become instability, the lower material parameters multiples is called strength reserve safety; Synthetical approach is the combination of over loading method and strength reserve method, it not only considers the several times upstream water load in the project we may encounter, but also considers the rock and weak structure face mechanical parameter lowing on the influence of stability, the multiples of overloading times and lowing strength times is called synthetical stability safety degree.

The merit of geomechanical model test is that it is an intuitive, perceptive analysis, it can get quite obviously visualize concept, macro and quantitative indicators of safety; it can get entire arch dam damage process, such as the formation of cracks and slip face and development until rupture; it can assume the slip face for other numerical methods and provide reference frame for loading measures. The shortcoming of geomechanical model test is that it is difficult to consider earthquake、seepage pressure and temperature changes and other load factors; it is difficult to completely simulate dam concrete and model material of dam foundation according to the similarity theory. The stress mechanism of high arch dam is very complicated. The model test is difficult to simulate affects of many nonlinear factors. The problems of rationality of analysis results quantitative estimates and model test technology are worth to study further.

Ⅳ. STABILITY SAFETY DEGREE UNDER OVER LOADING

Applying finite element numerical analysis method, dam body and foundation's stress field and displacement field are obtained. Adopting over loading or abasing material strength parameter, make the system reach the ultimate balanced state, thereby, over loading stability safety degree or strength reserve safety degree is received. The criterion which makes the system reaches the ultimate balanced state is called instability criterion. It belongs to elastic plastic ultimate balanced analysis category.

There are two possible reasons which bring on arch dam abutment and foundation rock come into plastic limit balanced state and cause instability. The first one is dam abutment and foundation rock design strength is close to actual strength, but dam meets with supernormal loads. Then we should adopt over loading method to analysis stability. The second one is under normal loading, but dam abutment and foundation holding force rock's design strength is differ from actual strength. Then we'd better adopt strength reserve method to analysis stability. When construction base face materials all give in yield and come into being slip ways, and some key points relative displacement on dam and construction base face is widened or inflexion appear at loading deformation curve, we believe that the arch dam shows the tendency to slip through construction base face, and the dam reaches the ultimate balanced state. This status is regarded as the instability criterion to arch dam.

Strength reserve method considers the material strength uncertainly and the possible deterioration, but rock characteristic of dam foundation is very complex in actual projects and it is difficult to correctly ascertain material intensity parameters, and which method is adopted to debase material strength parameters is yet to be further studied. Besides, debasing material strength parameters gradually is difficult to carry into execution by corresponding model test.

Ⅴ. STATIC CRITERIONS

5.1 Revision of conventional point anti-slide stability safely coefficient

The conventional evaluation stability methods of rock mass depend on the ratio of anti-slide force and sliding force, which is called anti-slide stability safety coefficient. The conventional point anti-slide stability safety coefficient computing formula is

$$k = (c - f\sigma_n)/\left|\tau_n\right|, \tag{2}$$

where $(c - f\sigma_n)$ is shear strength of the point on the investigated face, which is anti-slide force here; τ_n is shear stress which means sliding force.

The modal of i point anti-slide stability safely coefficient is shown in Fig. 2.

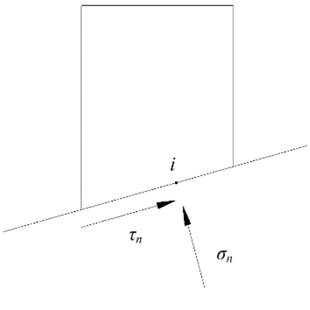

Fig.2 The modal of point anti-slide stability safely coefficient

When sliding face cannot be confirmed in advance, the minimum $k_{\min}$ of this point usually acts as the criterion to judge whether it is instability. If the angle between outer normal direction n and the first primary stress σ_1 is α, then σ_n and τ_n can be expressed by σ_1 and σ_3.

$$k_{\min} = \frac{c - f\left(\frac{\sigma_1+\sigma_3}{2} + \frac{\sigma_1-\sigma_3}{2}\cos 2\alpha\right)}{\left|\frac{1}{2}(\sigma_1-\sigma_3)\sin 2\alpha\right|} \tag{3}$$

Because of $\frac{\partial k}{\partial \alpha} = 0$, when k equals to $k_{\min}$, α is educed as

$$\alpha = \frac{1}{2}\arccos\left[\frac{\frac{1}{2}(\sigma_1-\sigma_3)f}{c-\frac{1}{2}(\sigma_1+\sigma_3)f}\right] \tag{4}$$

5.2 Revision of conventional point anti-slide stability safely coefficient

The definition and corresponding computation formula of the conventional evaluation methods about point anti-slide stability safely coefficient of rock is incompatible with mathematics and mechanics general knowledge.

Point anti-slide stability safely coefficient was denoted as Eq. (2), whose essential is the strength condition in classical mechanics rather than stability condition. $k_{\min}$ in Eq. (3) is shear strength safely coefficient of the point, which can not be used to judge the stability. The actual angle α on slip surface between outer normal direction n and the first primary stress σ_1 is not must be the value came from Eq. (4), so $k_{\min}$ calculated by Eq. (3) doesn't reflect the anti-slide stability degree of safety.

When calculate k by point σ_n and τ_n on actual slip surface as Eq. (2), the expression is incompatible with frictional theory and space vector algorithm. Firstly, anti-slide force $(c - f\sigma_n)$ and sliding force τ_n will be both vector. Secondly, it is known by slide frictional theory that the direction of anti-slide force is opposite to the relative slip direction of this point, while it may not be at the opposite direction to the slip force vector direction. Therefore, it is meaningless to compare the two different direction force vector like Eq. (2), and it breaks vector algorithm too.

In order to establish the anti-slid safety coefficient of the point according to the original meaning of Eq. (2) and following vector algorithm, the anti-slid force can only be compare with the component in the sliding direction $\mathbf{r}$, and the ratio can be defined as the point safety coefficient k_s along the direction $\mathbf{r}$, with this amendment, the point anti-slid stability safety coefficient formula is as Eq. (5).

$$k_s = (c - f\sigma_n)/\left|\tau_n\right|\cos\beta \tag{5}$$

In Eq. (5), β is the angle between composite shear stress direction τ_n of the point and sliding direction $\mathbf{r}$.

Anti-slide stability safety coefficient K_s is defined by the ratio of anti-slide force and sliding force on slip surface commonly, whose precondition is that slip surface is given in advance.

Based on the understanding about the questions above, frictional theory and vector geometry concept, anti-slide safety coefficient formula and the most dangerous composite sliding direction can be educed below.

$$K_s(\bar{r}) = \frac{\sum_{i=1}^{N_e}(c - f\sigma_{z'})_i A_i \sqrt{l_i'^2 + m_i'^2}}{\sum_{i=1}^{N_e}[l_i'\tau_{z'x'} + m_i'\tau_{z'y'}]} \tag{6}$$

In the equation, z' is the normal on some element slip surface, x' and y' are the two local coordinates in element slip surface tangential direction. l', m' and n' are the direction cosine of the angles between the body composite sliding direction $\bar{r}$ and local coordinate system x', y' and z' in element *i*. A_i is the slip surface acreage of element *i*. *Ne* is the total number of elements on slip surface.

Despite Eq. (6) gives out the formula of the new anti-slid stability safety coefficient but still maintain the framework of traditional definition of rock project. From the mechanics, its essential still has the meanings of strength checking, it extends to the reserve safety degree of the whole slip face shear strength from the concept of strength safety coefficient of a point in the mechanics. This is not the content of stability definition. In addition, from the Eq. (6) we can get the minimum safety value and the most disadvantaged slip direction, but how to determine the most disadvantaged possible slip face is still an outstanding problem.

Ⅵ. THE CRITERION OF DISTURBING ENERGY

According to the axioms of Dirichlet[6], the potential energy $\prod$ of the objects on the equilibrium configuration has the minimum value or the maximum value, if the $\prod$ is on the minimum value, the equilibrium configuration is stable, it's mathematical expression is $\delta\Pi = 0$, $\delta^2\Pi > 0$; if the $\prod$ is on the maximum value, the equilibrium is unstable, it's mathematical expression is $\delta^2\Pi < 0$;when the $\delta^2\Pi = 0$, the potential energy of the objects on the surface will not change after the minuteness deviation, and the equilibrium is random.

Based on the above ideas, the following relevant calculation formula can be established.

The Potential energy functional under the Lagrange system is as

$$\begin{aligned}\Pi &= \int_{\Omega} A(e_{ij})d\Omega - \int_{\Omega} f_i u_i d\Omega - \int_{s_\sigma} \overline{p_i} u_i ds \\ &= U - W\end{aligned} \tag{7}$$

In Eq. (7), the potential energy of the system is as

$$U = \int_{\Omega} A(e_{ij})d\Omega \tag{8}$$

The work of the external forces is as

$$W = \int_{\Omega} f_i u_i d\Omega + \int_{s_\sigma} \overline{p_i} u_i ds \tag{9}$$

The displacement of the equilibrium configuration is assumed as u_i^0 and the disturbing displacement is assumed as δu_i. The potential energy increment after disturbing is $\Delta\prod$. Based on the Taylor series expansion, $\Delta\prod$ can be expressed as

$$\begin{aligned}\Delta\prod &= \prod(u_i^0 + \delta u_i) - \prod(u_i^0) \\ &= \delta\prod + \frac{1}{2}\delta^2\prod + \cdots\end{aligned} \tag{10}$$

In Eq. (10), $\prod(u_i^0)$ is the total potential energy of inspection body in the original equilibrium position, $\prod(u_i^0 + \delta u_i)$ is the total potential energy of inspection body after disturbing displacement δu_i, $\Delta\prod$ is the disturbing energy caused by the disturbing displacement.

Because the total potential energy is stationary value before disturbing, that is $\Delta\prod = 0$. So there is the approximate relationship.

$$\Delta\prod = \frac{1}{2}\delta^2\prod \tag{11}$$

When do the numerical analyses, the value of $\delta^2\prod$ can be expressed by the potential energy increment $\Delta\prod$. The value of $\Delta\prod$ and $\delta^2\prod$ can also judge whether the inspecting body is stable or not, and the value of $\Delta\prod$ can be got by numerical computation.

According to references [7], the disturbance energy nonlinear expressions in the initial stability problem can be derived by using the finite element discrete.

$$\begin{aligned}\Delta\prod &= \sum_{e=1}^{N_e}\frac{1}{2}\{\Delta\delta^e\}^{\mathrm{T}}[k_{ep}]\{\Delta\delta^e\} \\ &+ \sum_{e=1}^{N_e}\frac{1}{2}\{\Delta\delta^e\}^{\mathrm{T}}[k_\sigma]\{\Delta\delta^e\} \\ &+ \sum_{e=1}^{N_e}\int_{\Omega^e}\{\Delta\delta^e\}^{\mathrm{T}}[B]\{\sigma^0\}d\Omega \\ &- \sum_{e=1}^{N_e}\{\Delta\delta^e\}^{\mathrm{T}}\{R_e^0\} \\ &= \Delta U - \Delta W\end{aligned} \tag{12}$$

In Eq. (12),

$$\begin{aligned}\Delta U = &\sum_{e=1}^{N_e}\frac{1}{2}\left\{\Delta\delta^e\right\}^{\mathrm{T}}\left[k_{ep}\right]\left\{\Delta\delta^e\right\}\\ &+\sum_{e=1}^{N_e}\frac{1}{2}\left\{\Delta\delta^e\right\}^{\mathrm{T}}\left[k_{\sigma}\right]\left\{\Delta\delta^e\right\}\\ &+\sum_{e=1}^{N_e}\int_{\Omega^e}\left\{\Delta\delta^e\right\}^{\mathrm{T}}\left[B\right]\left\{\sigma^0\right\}d\Omega\\ &-\sum_{e=1}^{N_e}\left\{\Delta\delta^e\right\}^{\mathrm{T}}\left\{R_e^0\right\}\end{aligned}\qquad(13)$$

$$\Delta W = \sum_{e=1}^{N_e}\left\{\Delta\delta^e\right\}^{\mathrm{T}}\left\{R_e^0\right\}\qquad(14)$$

In Eqs, (13) and (14), $\left\{\Delta\delta^e\right\}$ is the disturbing displacement. ΔU denotes the deformation energy increment which stores in the objects after disturbing, which is the factor to make the system revert to the initial position, calling disturbing internal energy. ΔW means the work that is outside force work on the disturbing displacement, which will consume the energy of the system.

According to references [7], the definition of stability safety coefficient is as

$$K_s = \frac{\Delta U}{\Delta W}\qquad(15)$$

Owing to the disturbing energy is scalar quantity, we can give the inspecting body's disturbing energy isoline after getting the disturbing energy of each unit of the inspection body. Obviously, the minimum disturbing energy isoline (face) which also has free face is the most dangerous latent slide face, and the most disadvantageous disturbing displacement of each point on the slip face's resultant displacement vector direction is the most disadvantageous slip direction.

Ⅶ. PROJECT EXAMPLE ANALYSIS

7.1 The stability analysis of high arch dam abutment

In the middle downstream of the Yalong River in china, a high arch dam is planed to be set up. The height of the dam crest is 1 885m, the height of construction base level is 1580m, the height of the dam 305m, ranking the first in the world. By applying the three-dimensional non-linear FEM theory and the disturbing energy method and static method of structural stability evaluation, this paper established the total three-dimensional computation model of the arch dam and foundation, and carried out a research for the total stability of arch dam and left abutment.

According to the geological conditions of left abutment, four groups possible slide mass can be identified. Due to lacking of information about initial stress, here, the initial stress field is the gravity stress field.

The stability safety coefficients of left abutment rock under natural conditions are given in Table 1.

TABLE 1, VALUES OF STABILITY SAFETY COEFFICIENT OF LEFT ABUTMENT ROCK

No.	(1)	(2)	(3)	(4)
disturbing energy method	1.721	1.867	1.795	1.816
static method	1.960	1.924	2.299	2.793

From the results shown in the table above, the four minimum clipping and friction stability safety factor against sliding of possible gliding mass is 1.924 of left dam abutment after the building of the dam, all in a stable condition, but security reserves are short.

7.2 Results comparison of several projects by using disturbing energy method

The values of the total stability safety coefficient of several arch dam in china by using disturbing energy method are given in Table 2.

TABLE 2, VALUES OF THE TOTAL STABILITY SAFETY COEFFICIENT OF SEVERAL ARCH DAMS

Names of projects	safety coefficient
Left Slope of Lijiaxia	1.72
Ertan	2.17
Xiaowan	1.88
Jingping	1.64
Xiluodu	2.60

Ⅷ. CONCLUSION

Based on the review and evaluation of the existing dam abutments and foundation and the total stability analysis method, this paper establishes the linking application of the disturbing energy method and the static method, the quantitative criterion standards and analysis of the local and total stability of arch dams, the criteria and method have been successfully applied to a number of projects. The disturbing energy method strictly abides by the basic principles of computational mechanics and gives the stability safety coefficient of local or total from the view of energy. Because the energy is a scalar, it avoids disunity of anti-slide stability safety coefficient leading by different projection methods. However, the safety coefficient got by the disturbing energy method is different from mechanism of anti-slide stability safety coefficient, the intrinsic link between them is still lack of deep study, and there isn't a stability criterion standard, they still can be a complement and reference. If combining with the disturbing energy method and the static method, especially it can be used to identify the latent slide mass and possible slide direction; combining with static method it can make comprehensive evaluation of the stability of dam abutment.

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